**Raaja Prabhu Uma Jaganathan**

**UIN: 824006931**

**CSCE 625 Assignment #6**

1. Translate the following sentences into First-Order Logic

* Tomatoes are either a fruit or vegetable.
  + ∀x tomato(x) 🡪 (fruit(x) ∨ vegetable(x))
* Every king has a crown and some subjects.­
  + ∀x king(x) 🡪 (hasCrown(x) ∧ hasSubject(x))
* An isosceles triangle is a triangle with 2 equal sides (but not 3).
  + ∀x,y,z isosceles(x,y,z) ↔ (eq(x, y) ∧ ¬eq(y,z)) ∨ (¬eq(x, y) ∧ eq(y,z)) ∨ (eq(x, z) ∧ ¬eq(y,z))
* Some mushrooms are poisonous.
  + ∃x mushroom(x) ∧ poisonous(x)
* Someone from the post-office is at the front door of John’s house.
  + ∃x person(x) ∧ worksAt(x, post-office) ∧ currentLocation(x, frontDoor(house(John)))
* All laptops sold by Dell in 2012 have at least 4 gigabytes of memory.
  + ∀x laptop(x) ∧ soldBy(x, Dell) ∧ during(sale(x), 2012) 🡪 greaterThanOREqual(memoryInGB(x), 4)

2. Following the procedure on pages 345-347 in the textbook (Sec. 9.5), convert the following sentences to CNF:

* b1,b2,c1,c2 contains(b1,c1)contains(b2,c2)  b1≠b2 → c1≠c2 (from Sammy's Sport Shop: "different boxes have different colors")
  + ¬contains(b1,c1)∨ ¬contains(b2,c2) ∨ ¬(b1≠b2) ∨ (c1≠c2)
* c country(c )∧ [∃x (tank(x) ∨ missle(x)) ∧ owns(c,x)] → militarized(c)
  + country(c )  tank(F(c))  owns(c,F(c))  militarized(c)
  + country(c )  missle(F(c))  owns(c,F(c))  militarized(c)
* x P(x)→ [ y P(y)→P(f(x,y)) ]  [y Q(x,y)→P(y) ]
  + P(x)  P(y) P(f(x,y))
  + P(x)  Q(x,z)
  + P(x)  P(z)

3. Determine whether or not the following pairs of predicates are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Terms that are variables are in capital letters; constants and function names are lowercase.

a) P(a, X, f(g(Y))) P(Z, f(Z), f(U))

Yes, the predicates are unifiable.

Θ = {Z|a, X|f(a), U|g(Y)}

P(a, f(a), f(g(Y))) P(a, f(a), f(g(Y)))

b) Q(f(a), g(X)) Q(Y, Y)

No, the predicates are not unifiable because Y has to correspond to two different functions, which cannot be unified (unless inverse of function g exists).

c) R(f(Y), Y,X) R(Z, f(a), f(V))

Yes, the predicates are unifiable.

Θ = {Y|f(a), Z|f(f(a)), X|f(V)}

R(f(f(a)), f(a), f(V)) R(f(f(a)), f(a), f(V))

d) P(a, Y, f(X)) P(Z, f(b), f(b))

Yes, the predicates are unifiable.

Θ = {Z|a, Y|f(b), X|b}

P(a, f(b), f(b)) P(a, f(b), f(b))

e) Q(g(f(a)), g(X), Z) Q(Y, Y, f(W))

Yes, the predicates are unifiable.

Θ = {Y|g(f(a)), X|f(a), Z|f(W)}

Q(g(f(a)), g(f(a)), f(W)) Q(g(f(a)), g(f(a)), f(W))

f) P(x,f(X),X) P(Y,f(a),b)

No, the predicates are not unifiable because X has to correspond to two different constants, which cannot be unified.

g) Q(f(a,a), V ,Z) Q(X, f(X,X), Y)

Yes, the predicates are unifiable.

Θ = {X|f(a,a), V|f(f(a,a),f(a,a)), Z|Y}

Q(f(a,a), f(f(a,a),f(a,a)), Y) Q(f(a,a), f(f(a,a),f(a,a)), Y)

4. Consider the following situation: *Marcus is a Pompeian. All Pompeians are Romans. Ceasar is a ruler. All Romans are either loyal to Caesar or hate Caesar (but not both). Everyone is loyal to someone. People only try to assassinate rulers they are not loyal to. Marcus tries to assassinate Caesar.*

1. Translate these sentences to First-Order Logic.

|  |  |  |
| --- | --- | --- |
| **#** | **English Sentence** | **Sentence in FoL** |
| 1 | Marcus is a Pompeian | Pompeian(Marcus) |
| 2 | All Pompeians are Romans | **x Pompeian(x) → Roman(x) |
| 3 | Ceasar is a ruler | Ruler(Ceasar) |
| 4 | All Romans are either loyal to Caesar or hate Caesar (but not both) | x Roman(x) → (loyal(x, Ceasar) hate(x, Ceasar)) (loyal(x, Ceasar) hate(x, Ceasar)) |
| 5 | Everyone is loyal to someone | x y loyal(x, y) |
| 6 | People only try to assassinate rulers they are not loyal to | x,y assassinate(x, y) → loyal(x, y) |
| 7 | Marcus tries to assassinate Caesar | assassinate(Marcus, Ceasar) |

b) Prove that ***Marcus hates Caesar*** using Natural Deduction. Label all derived sentences with prior sentences and unifier used.

|  |  |  |
| --- | --- | --- |
| **#** | **Deducted Sentence** | **Property, Unifier and sentences used** |
| 8 | loyal(Marcus, Ceasar) | Generalized Modus Ponen on 6 & 7  Θ = {x|Marcus, y|Ceasar} |
| 9 | Roman(Marcus) | Generalized Modus Ponen on 1 & 2  Θ = {x|Marcus} |
| 10 | (loyal(Marcus, Ceasar) hate(Marcus, Ceasar)) (loyal(Marcus, Ceasar) hate(Marcus, Ceasar)) | Generalized Modus Ponen on 4 & 9  Θ = {x|Marcus} |
| 11 | ((loyal(Marcus, Ceasar) hate(Marcus, Ceasar)) loyal(Marcus, Ceasar))  ((loyal(Marcus, Ceasar) hate(Marcus, Ceasar)) hate(Marcus, Ceasar)) | Distribute  over  in 10 |
| 12 | ((loyal(Marcus, Ceasar)loyal(Marcus, Ceasar)) hate(Marcus, Ceasar)loyal(Marcus, Ceasar)))  ((loyal(Marcus, Ceasar)hate(Marcus, Ceasar)) hate(Marcus, Ceasar)hate(Marcus, Ceasar))) | Distribute over in 11 |
| 13 | ((loyal(Marcus, Ceasar)loyal(Marcus, Ceasar)) hate(Marcus, Ceasar)loyal(Marcus, Ceasar))) |  Elimination on 12 |
| 14 | ((loyal(Marcus, Ceasar)hate(Marcus, Ceasar)) hate(Marcus, Ceasar)hate(Marcus, Ceasar))) |
| 15 | loyal(Marcus, Ceasar)hate(Marcus, Ceasar)) |  Elimination on 14 |
| 16 | hate(Marcus, Ceasar)hate(Marcus, Ceasar) |
| 17 | hate(Marcus, Ceasar)) | Resolution on 8 & 15 |

5. Represent the following information in FOL using Event Calculus and Interval Logic:

* A plant can only produce seeds after it has been polinated.
  + ∀x plant(x)  produceSeeds(x) 🡪 After(seedTime(x), polinateTime(x))
* The marching band of the home team plays during the halftime of all football games.
  + ∀x footballGame(x) 🡪plays(band(homeTeam(x))) during (playTime(band(homeTeam(x))), halftime(x))
* In any football game, if the score is tied at the end of the fourth quarter, there will be overtime.
  + ∀x footballGame(x) scoresEqual(end(4thQuarter(x))) 🡪 overtimeExists(x)